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Pattern of the Approximate Mass Degeneracy of Majorana Neutrinos

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Abstract

In view of the recently reported evidence for a nonzero Majorana mass of the electron neutrino, together with the established phenomena of atmospheric and solar neutrino oscillations, the case of three nearly mass-degenerate Majorana neutrinos is now a distinct possibility. I show in this paper how a natural pattern of symmetry breaking in the recently proposed A_4 model of Majorana neutrino masses can accommodate the data on neutrino oscillations, resulting in the predictions $\sin^2 2\theta_{atm} = 1$ and $\sin^2 2\theta_{sol} = 2/3$.

In the past several years, there has been mounting evidence for neutrino oscillations [1, 2, 3]. Since they require only neutrino mass differences, the possibility of nearly degenerate neutrino masses is often considered [4]. Recently, the first positive evidence for neutrinoless double beta decay has been reported [5] which may be interpreted as an effective nonzero Majorana mass for the electron neutrino. Combined with the atmospheric and solar neutrino data, there is now a plausible argument for three nearly mass-degenerate Majorana neutrinos [6]. However, the charged-lepton masses are certainly not degenerate, so whatever symmetry is used to maintain the neutrino mass degeneracy must be broken. To implement this idea in a renormalizable field theory, the discrete symmetry A_4 was proposed recently [7] where its spontaneous breaking results in charged-lepton masses and its explicit soft breaking results in neutrino mass differences.

The proposed A_4 model is based on a simple model of neutrino masses [8], where a leptonic Higgs doublet $\eta = (\eta^+, \eta^0)$ and three right-handed neutral singlet fermions N_{iR} are added to the minimal standard model of particle interactions. These new particles may all be at or below the TeV energy scale, so that the seesaw mechanism [9] may be tested experimentally at future accelerators. With the report of a possible discrepancy in the experimental measurement [10] of the muon anomalous magnetic moment with the theoretical prediction, this leptonic Higgs model was used [11] to constrain the masses and couplings of these new particles. However, a sign error has been discovered [12] in the theoretical calculation, hence the experimental discrepancy is now only 1.6σ, which is not much of a constraint on this model.

In this paper the explicit soft breaking of the A_4 symmetry is shown to allow for a natural solution with the predictions $\sin^2 2\theta_{atm} = 1$ and $\sin^2 2\theta_{sol} = 2/3$ which agree well with present data and may be tested more precisely in future neutrino-oscillation experiments. At the same time, the new particles N_{iR} as well as an assortment of Higgs bosons with specific properties [7] are predicted to be accessible at future high-energy accelerators and their decays into leptons will map out the neutrino mass matrix.

Under A_4 and L (lepton number), the color-singlet fermions and scalars of this model transform as follows.

$$(\nu_i, l_i)_L \sim (\underline{3}, 1), \tag{1}$$

$$l_{1R} \sim (\underline{1}, 1), \tag{2}$$

$$l_{2R} \sim (\underline{1}', 1), \tag{3}$$

$$l_{3R} \sim (\underline{1}'', 1), \tag{4}$$

$$N_{iR} \sim (\underline{3}, 0), \tag{5}$$

$$\Phi_i = (\phi_i^+, \phi_i^0) \sim (\underline{3}, 0), \tag{6}$$

$$\eta = (\eta^+, \eta^0) \sim (\underline{1}, -1). \tag{7}$$

Hence its Lagrangian has the invariant terms

$$\frac{1}{2} M N_{iR}^2 + f \bar{N}_{iR}(\nu_{iL}\eta^0 - l_{iL}\eta^+) + h_{ijk}\overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + h.c., \tag{8}$$

where

$$h_{i1k} = h_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad h_{i2k} = h_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix}, \quad h_{i3k} = h_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix}, \tag{9}$$

with $\omega^3 = 1$. Thus the neutrino mass matrix (in this basis) is proportional to the unit matrix with magnitude $f^2 u^2 / M$, where $u = \langle \eta^0 \rangle$, whereas the charged-lepton mass matrix is given by

$$\mathcal{M}_l = \begin{bmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2 \omega v_2 & h_3 \omega^2 v_2 \\ h_1 v_3 & h_2 \omega^2 v_3 & h_3 \omega v_3 \end{bmatrix}, \tag{10}$$

where $v_i = \langle \phi_i^0 \rangle$. Now rotate $\mathcal{M}_l$ on the left by

$$U_L^\dagger = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}, \tag{11}$$

then

$$U_L^\dagger \mathcal{M}_l = \begin{bmatrix} v & v' & v'' \\ v'' & v & v' \\ v' & v'' & v \end{bmatrix} \begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{bmatrix}, \tag{12}$$

where

$$v = \frac{1}{\sqrt{3}}(v_1 + v_2 + v_3), \tag{13}$$

$$v' = \frac{1}{\sqrt{3}}(v_1 + \omega v_2 + \omega^2 v_3), \tag{14}$$

$$v'' = \frac{1}{\sqrt{3}}(v_1 + \omega^2 v_2 + \omega v_3). \tag{15}$$

As shown in Ref. [7], $v_1 = v_2 = v_3$ is a natural solution of the A_4-symmetric Higgs potential, in which case $v' = v'' = 0$ and $U_L^\dagger \mathcal{M}_l$ is diagonal. Hence

$$\mathcal{M}_\nu = \frac{f^2 u^2}{M} U_L^T U_L = \frac{f^2 u^2}{M} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \tag{16}$$

in the $(\nu_e, \nu_\mu, \nu_\tau)$ basis. This shows that ν_μ mixes maximally with ν_τ, but since all physical neutrino masses are degenerate, there are no neutrino oscillations. To break the degeneracy, let $\mathcal{M}_{ij} = M\delta_{ij} + m_{ij}$, then

$$(\mathcal{M}^{-1})_{ij} \simeq M^{-1}\delta_{ij} - M^{-2} m_{ij}. \tag{17}$$

Whereas m_{ij} is assumed arbitrary in Ref. [7], it is required here to be invariant under U_L, i.e.

$$U_L^T m_{ij} U_L = m_{ij}. \tag{18}$$

It is then a simple exercise to show that the most general solution is of the form

$$m_{ij} = \begin{bmatrix} 2\delta + 2\delta' & \delta' & \delta' \\ \delta' & \delta & \delta \\ \delta' & \delta & \delta \end{bmatrix}. \tag{19}$$

Consider first the case $\delta' = 0$, then

$$\mathcal{M}_\nu \simeq \frac{f^2u^2}{M}\begin{bmatrix} 1-2\delta/M & 0 & 0 \\ 0 & -\delta/M & 1-\delta/M \\ 0 & 1-\delta/M & -\delta/M \end{bmatrix}, \tag{20}$$

which has eigenvalues proportional to $1-2\delta/M$, $1-2\delta/M$, and -1, corresponding to the eigenstates ν_e, $(\nu_\mu+\nu_\tau)/\sqrt{2}$, and $(\nu_\mu-\nu_\tau)/\sqrt{2}$ respectively. This shows that the threefold degeneracy of $\mathcal{M}_\nu$ is broken by δ to a twofold degeneracy with $\nu_\mu-\nu_\tau$ maximal mixing and $\Delta m^2 \simeq 4\delta f^4u^4/M^3$, which is desirable for explaining atmospheric neutrino oscillations [1]. It also provides a natural reason for having $\delta' << \delta$ because δ' breaks even the twofold degeneracy, as discussed below.

To see how $\delta' \neq 0$ affects m_{ij} of Eq. (19), rotate $\mathcal{M}_\nu$ of Eq. (20) to the basis spanned by ν_e, $(\nu_\mu+\nu_\tau)/\sqrt{2}$, and $(\nu_\mu-\nu_\tau)/\sqrt{2}$. Then

$$\mathcal{M}_\nu \simeq \frac{f^2u^2}{M}\begin{bmatrix} 1-2\delta/M-2\delta'/M & -\sqrt{2}\delta'/M & 0 \\ -\sqrt{2}\delta'/M & 1-2\delta/M & 0 \\ 0 & 0 & -1 \end{bmatrix}, \tag{21}$$

which has the solution

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta/\sqrt{2} & \sin\theta/\sqrt{2} \\ -\sin\theta & \cos\theta/\sqrt{2} & \cos\theta/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix}, \tag{22}$$

where $\tan\theta = (\sqrt{3}-1)/\sqrt{2}$, and

$$m_1 \simeq \frac{f^2u^2}{M}\left[1-\frac{2\delta}{M}-\frac{(\sqrt{3}+1)\delta'}{M}\right], \tag{23}$$

$$m_2 \simeq \frac{f^2u^2}{M}\left[1-\frac{2\delta}{M}+\frac{(\sqrt{3}-1)\delta'}{M}\right], \tag{24}$$

and $m_3 \simeq -f^2u^2/M$. Hence

$$\Delta m^2_{12} \simeq \frac{4\sqrt{3}\delta' f^4u^4}{M^3}, \quad \sin^2 2\theta_{12} = \frac{2}{3}. \tag{25}$$

This is then a satisfactory explanation of the solar neutrino data [2] with a large mixing angle. Numerically, let the common mass of all three neutrinos be $f^2u^2/M = 0.4$ eV [5], and $\delta/M = 3.9 \times 10^{-3}$, $\delta'/M = 3.6 \times 10^{-5}$; then $(\Delta m^2)_{atm} = 2.5 \times 10^{-3}$ eV^2 and $(\Delta m^2)_{sol} = 4.0 \times 10^{-5}$ eV^2, in good agreement with present data. Note also that these numbers are not spoiled by radiative corrections [6].

As Eq. (22) shows, under the assumption of $v_1 = v_2 = v_3$ and that of Eq. (18), the electron neutrino ν_e has only ν_1 and ν_2 components, i.e. $U_{e3} = 0$. This is perfectly consistent with present data. However, if U_{e3} is indeed zero, then there can be no CP-violating effects in neutrino oscillations. In the context of the present model, if the Higgs potential has soft terms which break the A_4 symmetry, then v' and v'' of Eqs. (14) and (15) will not be zero, but may be assumed to be small. In that case, (h_1, h_2, h_3) of Eq. (12) are still approximately proportional to (m_e, m_μ, m_τ), and it is easy to show that the rotation due to $v' \neq 0$ and $v'' \neq 0$ results in $|U_{e3}| \simeq |v'/v\sqrt{2}|$, which is bounded by reactor experiments [13] to be less than about 0.16.

The heavy right-handed singlet fermions N_{iR} are of course also nearly degenerate in mass. As discussed in Ref. [8], they will decay into charged leptons plus either a $W^\pm$ boson or a charged Higgs boson. The mass eigenstates of N_{iR} are given by $N_1 \cos\theta + (N_2 + N_3)\sin\theta/\sqrt{2}$, $-N_1 \sin\theta + (N_2 + N_3)\cos\theta/\sqrt{2}$, and $(N_2 - N_3)/\sqrt{2}$, with masses $M + 2\delta + (\sqrt{3}+1)\delta'$, $M + 2\delta - (\sqrt{3}-1)\delta'$, and M respectively. Their couplings are given by Eqs. (8) and (11), i.e. $N_1 \to (e + \mu + \tau)/\sqrt{3}$, $N_2 \to (e + \omega\mu + \omega^2\tau)/\sqrt{3}$, and $N_3 \to (e + \omega^2\mu + \omega\tau)/\sqrt{3}$. An equal and incoherent mixture of all three N's will of course decay equally into e, μ, and τ.

In conclusion, the case of three nearly mass-degenerate Majorana neutrinos in a renormalizable field theory based on the discrete symmetry A_4 is studied and found to accommodate a natural solution with two mass splittings, one larger than the other because it breaks the threefold degeneracy only down to a twofold degeneracy. This pattern is ideal for understanding the recently reported evidence for a nonzero effective Majorana mass of the electron neutrino, and the established phenomena of atmospheric and solar neutrino oscillations. It predicts $\sin^2 2\theta_{atm} = 1$ and $\sin^2 2\theta_{sol} = 2/3$ with a zero or small U_{e3}. It also predicts particles at the TeV energy scale which are responsible for the proposed pattern. As such, future neutrino-oscillation experiments are complementary to future high-energy accelerator experiments in the unambiguous test of this model.

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